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Multiple-Symbol Differential Detection for Spatial Modulation

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Abstract– Approximately 3 dB Signal-to-Noise Ratio (SNR) loss is always paid with conventional Differential Spatial Modulation (DSM) as compared to coherent Spatial Modulation (SM). In this paper, a Multiple-Symbol Differential Detection (MSDD) technique is proposed for DSM systems to mitigate the loss due to differential detection. The new scheme can greatly narrow the 3 dB performance gap by extending the observation interval for differential decoding. The technique uses maximum-likelihood sequence detection instead of traditional symbol-by-symbol detection, and is carried out on the slow, flat Rayleigh fading channel. A generalized decision metric is derived for an observation interval of arbitrary length. It is shown that for a moderate number of symbols, MSDD provides approximately 1.5 dB performance improvement over the conventional differential detection. In addition, a closed-form pairwise error probability and approximate bit error probability (BEP) are derived for multiple-symbol differential spatial modulation. Results show that the theoretical BEP matches well the simulated one. The BEP is shown to converge asymptotically with the number of symbols in the observation interval to that of the differential scheme with coherent detection.

Keywords– MIMO, Differential Spatial Modulation, Multiple-Symbol differential detection (MSDD), decision metric, pairwise error probability (PEP).

1 INTRODUCTION

In the past few years, SM [1–5] and SSK [6] have been receiving increasing research interest. The key idea of SM is to activate one among n_T transmit antennas for data transmission at a given symbol instant. By exploiting the difference in the channel impulse responses of transmit-to-receive wireless links, this arrangement enables the use of the transmit antenna index as an information carrying means, in addition to the transmitted symbol chosen from a constellation diagram. Consequently, the entire spectral efficiency can be increased by the base-two logarithm of the number of transmit antennas [2]. The so-called SSK scheme can be seen as a special case of SM, where the transmitted symbol is always the same, and the antenna index is the only information carrying unit. Compared to other Multiple Input Multiple Output (MIMO) techniques, such as the Vertical Bell Laboratories Layered Space-Time (V-BLAST) [7] and Space-Time Block Codes (STBCs) [8, 9], SM and SSK have several appealing characteristics, which are brought about by the following two key features: 1) only one antenna remains active during transmission and 2) the spatial position (i.e., the index) of each transmit antenna in the antenna array is used to convey information. Unlike V-BLAST, in which joint detection is required to mitigate the effect of Inter-Channel Interference (ICI) in order to achieve a high multiplexing gain, SM and SSK can entirely avoid ICI, thus allowing low-complexity single-stream detectors to be implemented at the receiver. Unlike STBCs, which were developed to attain the maximum diversity order,

SM and SSK can introduce a multiplexing gain that increases logarithmically with the number of transmit antennas. Furthermore, SM and SSK do not require the Inter-Antenna Synchronization (IAS) to be as strict as V-BLAST and STBCs do. As a consequence, it has been concluded in [2, 3] and [6] that SM/SSK is a promising MIMO technique that has the potential to outperform conventional MIMO techniques.

Until now, most investigations of SM/SSK have assumed that perfect channel state information is available at the receiver. This assumption is reasonable when the channel changes slowly in comparison with the symbol transmission rate, whereas in high-mobility situations, the above assumption is questionable. In such situations, the channel changes rapidly, and hence it is costly and difficult to obtain exact channel state information. Therefore, a very popular and widely accepted way to deal with the system design for high mobility scenarios is to avoid the need of the channel state information by using differential signaling. The differential concept has been widely used in the design of many technologies for high rate systems. As mentioned in [10], the differential concept has been successfully implemented in MIMO, e.g., the differential Alamouti scheme [11] and differential spatial multiplexing [12]. However, it has not been used with the SM/SSK technology. This is because, unlike other MIMO technologies, the channel in SM/SSK systems actually acts as a modulation unit, which makes the design of differential SM/SSK unique and difficult. Recently, in [13], a differential transmission scheme has been proposed for Space-Time Shift Keying system

where, as addressed by the authors, the concept of SM/SSK is extended to include both the space and time dimensions. However, in this scheme multiple transmit antennas remain active at each symbol duration. This means that the scheme in [13] does not treat the channel as the actual modulation unit. As clearly mentioned in [10], properly designing differential SM/SSK remains a big challenge and is still an open issue. Very recently, the authors in [14] have proposed a differential scheme for SM, called Differential SM (DSM), which can be applied to any equal energy signal constellations. The developed DSM retains the key feature of SM in that only one antenna is active at any symbol instant. Therefore, ICI is avoided and the requirement of IAS is relaxed. However, similar to other differential schemes, a loss of approximately 3 dB in the SNR is always observed in this differential scheme.

The MSDD was first introduced for additive white Gaussian noise (AWGN) channels by Divsalar and Simon [15]. By extending the observation interval to more than two symbols, the technique makes use of Maximum Likelihood (ML) sequence detection instead of symbol-by-symbol detection, as in the conventional differential detection. The performance of MSDD depends on the number of observed symbols. For a moderate number of symbols, MSDD bridges the performance gap between noncoherent and coherent communications. In [16] and [17], MSDD was applied to the flat Rayleigh fading channel. Motivated by MSDD, the observation interval of the differential Alamouti STBC is extended to three blocks [18]. As a result, a performance improvement of about 0.5 dB for binary phase-shift keying (BPSK) messages was demonstrated. In [19], the MSDD scheme for space-time blocks incorporates knowledge of the fading correlation.

In this paper, we generalize the differential SM to a larger number of observation intervals. The decision metric for a sequence of L observed blocks is derived. It is shown that for an observation interval of $L = 8$ blocks, there is a gain of about 1.5 dB over differential SM with two blocks.

The remaining of the paper is organized as follows. We briefly overview the principle of DSM in Section 2. In Section 3, the generalized decision metric is derived for DSM with an observation interval of L blocks over the slow Rayleigh fading channel. It is shown that special case $L = 2$ coincides with the results in [14]. In Section 4, the closed-form pairwise error probability (PEP) is derived, and an approximate bit-error probability (BEP) for this scheme is analyzed. Finally, conclusions are drawn in Section 5.

Notations: Throughout the paper we use the following mathematical notations. The bold small/capital letter denotes vector/matrix, respectively. $(\cdot)^H$ is used for Hermitian operation while $\|\cdot\|_F$ for the Frobenius norm of a vector or matrix. $E[\Delta]$ and $\text{Var}[\Delta]$ denote the statistical expectation and variance of Δ , respectively. $\text{trace}[\cdot]$ represents the trace function and $\text{diag}\{\cdot\}$ creates a diagonal matrix whose diagonal entries taken values from $\{\cdot\}$. $\det \Lambda$ is determinant of Λ matrix. We use $\text{Re}[a]$ for the real part of the complex number a .

2 DSM SYSTEM MODEL

We consider a MIMO communication system equipped with n_T transmit antennas and n_R receive antennas. In order to avoid the need for channel state information at the receiver, Yuyang Bian *et al.* proposed a DSM scheme for the case of two transmitter antennas [14]. In DSM, the transmitted signal vectors over two adjacent time intervals are collected into an $n_T \times T$ transmitted signal matrix \mathbf{S}_t , where $n_T = 2$ and $T = 2$. Let \mathbf{H}_t be the $n_R \times n_T$ fading matrix with the (i, j) -th entry, $h_{i,j}$, denoting the normalized complex fading gain from transmit antenna j to receive antenna i . The received signal at time intervals t can be expressed as

$$\mathbf{Y}_t = \mathbf{H}_t \mathbf{S}_t + \mathbf{N}_t, \quad (1)$$

where \mathbf{Y}_t is the $n_R \times T$ received signal matrix and \mathbf{N}_t is the $n_R \times T$ AWGN matrix. The entries of \mathbf{H}_t are assumed to be independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance, whereas those of \mathbf{N}_t are assumed to be i.i.d. complex Gaussian random variables with zero mean and variance of N_0 .

The transmitted signal matrix \mathbf{S}_t satisfies the condition that each transmit antenna is activated only once. Hence, \mathbf{S}_t can be a diagonal or anti-diagonal matrix. Furthermore, to enable differential transmission, the signal constellation is restricted to an equal energy constellation, i.e. M_0 -PSK, in particular.

The transmission signal space Ω_{DSM} of DSM is given by

$$\Omega_{\text{DSM}} = \left\{ \begin{bmatrix} s_1 & 0 \\ 0 & s_2 \end{bmatrix}, \begin{bmatrix} 0 & s_1 \\ s_2 & 0 \end{bmatrix} \mid s_1, s_2 \in \mathbf{A} \right\}, \quad (2)$$

where \mathbf{A} is the M_0 -PSK constellation. The element $\begin{bmatrix} s_1 & 0 \\ 0 & s_2 \end{bmatrix}$ decides that at time interval 1, the symbol s_1 chosen from \mathbf{A} is transmitted over the transmit antenna 1 meanwhile the transmit antenna 2 remains idle. And at time interval 2, the symbol s_2 chosen from \mathbf{A} is transmitted by the antenna 2 meanwhile the transmit antenna 1 remains idle. The matrix $\begin{bmatrix} 0 & s_1 \\ s_2 & 0 \end{bmatrix}$ is also interpreted in a similar way.

Because of using the transmission signal space Ω_{DSM} formed by M_0 -PSK constellation, we can map $\log_2 M_0$ bits to the first symbol s_1 , another $\log_2 M_0$ bits to the second symbol s_2 , and the left 1 bit to the active index of the transmit antennas. The spectral efficiency of DSM can be achieved as

$$\eta_{\text{DSM}(n_T=2)} = 0.5 + \log_2 M_0 \text{ bpcu}. \quad (3)$$

The block diagram of the DSM transmitter is given in Figure 1. The transmitter begins the differential

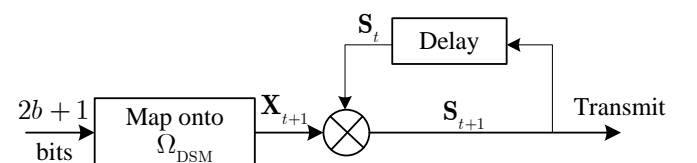


Figure 1. Block diagram of differential SM transmitter.

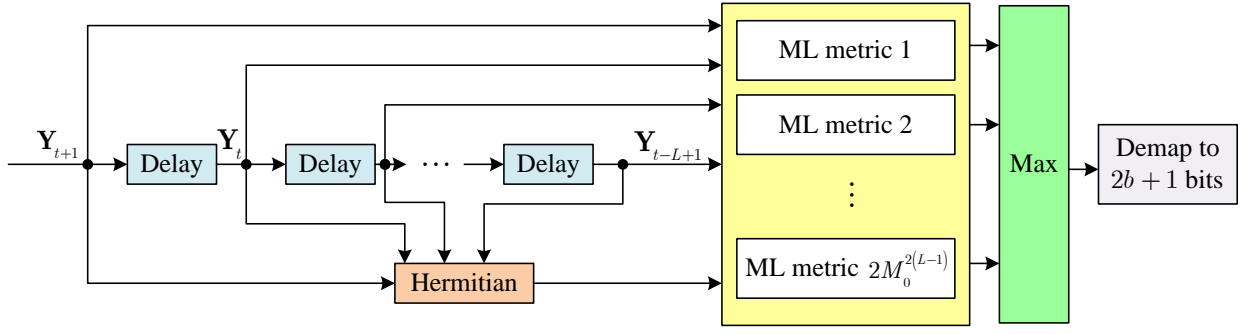


Figure 2. Block diagram of multiple-symbol differential SM receiver.

transmission process by sending an arbitrary matrix $\mathbf{S}_0 \in \Omega_{\text{DSM}}$ which may be unknown at the receiver. Note that this transmitted signal matrix does not convey any information. The encoding words are created as follows. At time interval $2t+2$, map a group of $2b+1 = 2\log_2 M_0 + 1$ bits into a codeword matrix $\mathbf{X}_{t+1} \in \Omega_{\text{DSM}}$. Compute the actual transmitted signal matrix \mathbf{S}_{t+1} as

$$\mathbf{S}_{t+1} = \mathbf{S}_t \mathbf{X}_{t+1}. \quad (4)$$

The transmitted signal matrix \mathbf{S}_{t+1} is sent at time intervals $2t+2$ and $2t+3$. This process is repeated until the end of the transmission.

At the receiver side, the received matrix at time intervals $2t+2$ and $2t+3$ is given by

$$\begin{aligned} \mathbf{Y}_{t+1} &= \mathbf{H}_{t+1} \mathbf{S}_{t+1} + \mathbf{N}_{t+1} \\ &= \mathbf{H}_{t+1} \mathbf{S}_t \mathbf{X}_{t+1} + \mathbf{N}_{t+1} \\ &= \mathbf{Y}_t \mathbf{X}_{t+1} - \mathbf{N}_t \mathbf{X}_{t+1} + \mathbf{N}_{t+1}. \end{aligned} \quad (5)$$

To get the transmitted information bits, we need to estimate \mathbf{X}_{t+1} . The optimal ML detector can be derived as

$$\begin{aligned} \hat{\mathbf{X}}_{t+1} &= \arg \min_{\mathbf{X} \in \Omega_{\text{DSM}}} \|\mathbf{Y}_{t+1} - \mathbf{Y}_t \mathbf{X}\|_F^2 \\ &= \arg \max_{\mathbf{X} \in \Omega_{\text{DSM}}} \text{trace}\{\text{Re}(\mathbf{Y}_{t+1}^H \mathbf{Y}_t \mathbf{X})\}. \end{aligned} \quad (6)$$

According to (6), the block diagram of the receiver is designed as shown in Figure 2, where $L=2$ corresponds to the case of the conventional DSM detection.

3 PROPOSED MSDD FOR DSM SYSTEM

Consider a DSM system in which the transmitter is presented in Figure 1. In order to estimate the transmitted DSM symbols, we have presented the principle of the DSM receiver using the ML detection as given in (6). In this section, we propose a new DSM receiver using MSDD. The configuration of the proposed receiver is shown in Figure 2. Different from the conventional DSM detection, the MSDD receiver extends the observation interval to $L > 2$ blocks. In the proposed receiver, for M_0 -PSK modulation, the receiver needs to compute and compare $2M_0^{2(L-1)}$ decision metrics as illustrated in Figure 2. These metrics will be derived as follows. Assume that the channel stays constant during the

observation interval. Then the received signal sequence can be expressed as

$$\mathbf{Y} = \mathbf{H}\mathbf{S} + \mathbf{N}, \quad (7)$$

where

$$\begin{aligned} \mathbf{Y} &= [\mathbf{Y}_t, \mathbf{Y}_{t-1}, \dots, \mathbf{Y}_{t-L+1}] \\ \mathbf{S} &= \text{diag}\{\mathbf{S}_t, \mathbf{S}_{t-1}, \dots, \mathbf{S}_{t-L+1}\} \\ \mathbf{H} &= [\mathbf{H}_t, \mathbf{H}_{t-1}, \dots, \mathbf{H}_{t-L+1}] \\ \mathbf{N} &= [\mathbf{N}_t, \mathbf{N}_{t-1}, \dots, \mathbf{N}_{t-L+1}]. \end{aligned}$$

The matrices \mathbf{Y} , \mathbf{H} and \mathbf{N} are of size $2 \times 2L$ and \mathbf{S} of size $2L \times 2L$. For convenience, we also define a $2L \times 2L$ matrix $\mathbf{X} = \text{diag}\{\mathbf{X}_t, \mathbf{X}_{t-1}, \dots, \mathbf{X}_{t-L+1}\}$.

Consider the channel \mathbf{H} is fixed over some time interval. For a block of L observations, conditioned on \mathbf{S} , the received matrix \mathbf{Y} is a zero-mean Gaussian-distributed matrix. Its conditional pdf (probability density function) is

$$p(\mathbf{Y}|\mathbf{S}) = \frac{1}{(\pi)^{4L} \det \mathbf{\Lambda}} \exp \left\{ -\text{trace} \left(\mathbf{Y}^H \mathbf{\Lambda}^{-1} \mathbf{Y} \right) \right\}, \quad (8)$$

where $\mathbf{\Lambda}$ is the covariance matrix of \mathbf{Y} and is given by $\mathbf{\Lambda} = E\{\mathbf{Y}\mathbf{Y}^H|\mathbf{S}\}$. Since path gains are assumed constant during a frame, $\mathbf{H}_i = \mathbf{H}_j$ for all $i \neq j$, we have

$$\begin{aligned} \mathbf{\Lambda} &= E[(\mathbf{H}\mathbf{S} + \mathbf{N})(\mathbf{H}\mathbf{S} + \mathbf{N})^H] \\ &= E[\mathbf{S}\mathbf{H}\mathbf{H}^H\mathbf{S}^H + \mathbf{N}\mathbf{N}^H] \\ &= \mathbf{S}(\mathbf{I}_{n_T} \otimes \mathbf{1}_L)\mathbf{S}^H + N_0\mathbf{I}_{2L}, \end{aligned} \quad (9)$$

where $\mathbf{1}_L$ denotes an $L \times L$ matrix with all elements equal to one, and \otimes represents the Kronecker product.

Using the unitary property of the matrix \mathbf{S} , it can be shown that $\det \mathbf{\Lambda}$ is independent of the messages $\mathbf{X}_t, \mathbf{X}_{t-1}, \dots, \mathbf{X}_{t-L+1}$.

Let us define \mathbf{V} , \mathbf{Z} and \mathbf{X} as

$$\begin{aligned} \mathbf{V} &= N_0\mathbf{I}_{2L}, \\ \mathbf{Z} &= \mathbf{S}(\mathbf{I}_2 \otimes \mathbf{1}_{L \times 1}), \\ \mathbf{D} &= (\mathbf{I}_2 \otimes \mathbf{1}_{L \times 1})^H \mathbf{S}^H. \end{aligned} \quad (10)$$

Then (9) can be expressed as $\mathbf{\Lambda} = \mathbf{Z}\mathbf{I}_{2L}\mathbf{D} + \mathbf{V}$. Using the matrix inversion lemma [20]

$$(\mathbf{V} + \mathbf{Z}\mathbf{I}_{2L}\mathbf{D})^{-1} = \mathbf{V}^{-1} - \mathbf{V}^{-1}\mathbf{Z}(\mathbf{I}_{2L}^{-1} + \mathbf{D}\mathbf{V}^{-1}\mathbf{Z})\mathbf{D}\mathbf{V}^{-1}, \quad (11)$$

and (9) yields

$$\mathbf{\Lambda}^{-1} = \frac{1}{N_0}\mathbf{I}_{2L} - \frac{1}{N_0(1+N_0)}\mathbf{S}(\mathbf{I}_2 \otimes \mathbf{1}_L)\mathbf{S}^H. \quad (12)$$

Since the natural logarithm is a monotonically increasing function of its argument, maximizing $p(\mathbf{Y}|\mathbf{X})$ qua \mathbf{X} in (8) is equivalent to maximizing $\ln p(\mathbf{Y}|\mathbf{X})$. Choosing the sequence \mathbf{X} to maximize (8) results in the decision metric

$$\begin{aligned}\hat{\xi} &= \text{trace} \left[-\ln(\det \Lambda) - (\mathbf{Y}^H \Lambda^{-1} \mathbf{Y}) \right] \\ &= \text{trace} \left[-\ln(\det \Lambda) - \frac{1}{N_0} \mathbf{Y}^H \mathbf{Y} \right. \\ &\quad \left. + \frac{1}{N_0(1+N_0)} \mathbf{Y}^H (\mathbf{S} (\mathbf{I}_2 \otimes \mathbf{1}_L) \mathbf{S}^H) \mathbf{Y} \right].\end{aligned}\quad (13)$$

As $\det \Lambda$, $\mathbf{Y}^H \mathbf{Y}$, N_0 are independent of transmitted messages, they can be ignored. Then the decision metric becomes

$$\tilde{\xi} = \text{trace} \left[\mathbf{Y}^H \mathbf{S} (\mathbf{I}_2 \otimes \mathbf{1}_L) \mathbf{S}^H \mathbf{Y} \right]. \quad (14)$$

Expanding (14), the metric can be expressed as

$$\begin{aligned}\tilde{\xi} &= \text{trace} \left[\sum_{i=0}^{L-1} \sum_{j=0}^{L-1} \mathbf{Y}_{t-i}^H \mathbf{S}_{t-i} \mathbf{S}_{t-j}^H \mathbf{Y}_{t-j} \right] \\ &= K + 2 \times \text{trace} \left\{ \text{Re} \left(\sum_{i=1}^{L-1} \sum_{j=0}^{i-1} \mathbf{Y}_{t-i}^H \mathbf{S}_{t-i} \mathbf{S}_{t-j}^H \mathbf{Y}_{t-j} \right) \right\},\end{aligned}\quad (15)$$

where

$$K = \text{trace} \left[\sum_{i=0}^L \mathbf{Y}_{t-i}^H \mathbf{S}_{t-i} \mathbf{S}_{t-j}^H \mathbf{Y}_{t-j} \right], \quad (16)$$

Due to the unitary property of \mathbf{S}_{t-i} , K is independent of the transmitted symbol sequence. Thus the decision metric becomes

$$\tilde{\xi} = \text{trace} \left[\text{Re} \left(\sum_{i=1}^{L-1} \sum_{j=0}^{i-1} \mathbf{Y}_{t-i}^H \mathbf{S}_{t-i} \mathbf{S}_{t-j}^H \mathbf{Y}_{t-j} \right) \right]. \quad (17)$$

Using the identity for the trace function [20], we have

$$\begin{aligned}\tilde{\xi} &= \text{trace} \left\{ \text{Re} \left[\sum_{i=1}^{L-1} \sum_{j=0}^{i-1} \mathbf{Y}_{t-j} \mathbf{Y}_{t-i}^H \mathbf{S}_{t-i} \mathbf{S}_{t-j}^H \right] \right\} \\ &= \text{trace} \left\{ \text{Re} \left[\sum_{i=1}^{L-1} \sum_{j=0}^{i-1} \mathbf{Y}_{t-j} \mathbf{Y}_{t-i}^H (\mathbf{X}_{t-j} \dots \mathbf{X}_{t-i+1})^H \right] \right\}.\end{aligned}\quad (18)$$

The differentially encoded message $\hat{\mathbf{X}}$ can be detected from

$$\begin{aligned}\hat{\mathbf{X}} &= \arg \max_{\mathbf{C}} \text{trace} \\ &\quad \left\{ \text{Re} \left[\sum_{i=1}^{L-1} \sum_{j=0}^{i-1} \mathbf{Y}_{t-j} \mathbf{Y}_{t-i}^H (\mathbf{X}_{t-j} \mathbf{X}_{t-j-1} \dots \mathbf{X}_{t-i+1})^H \right] \right\}.\end{aligned}\quad (19)$$

This is the MSDD decision metric for an observation interval of L blocks. The complexity of the MSDD receiver increases exponentially with the length of the observation interval.

Next, we discuss the special cases of $L = 2$ and $L = 3$.

1) $L = 2$: The detected message in (19) becomes

$$\hat{\mathbf{X}}_t = \arg \max_{\mathbf{X}} \text{trace} \left[\text{Re} \left(\mathbf{Y}_t \mathbf{Y}_{t-1}^H \mathbf{X}_t^H \right) \right]. \quad (20)$$

2) $L = 3$: We can detect $\mathbf{X}_t, \mathbf{X}_{t-1}$ by

$$\begin{aligned}[\hat{\mathbf{X}}_t \hat{\mathbf{X}}_{t-1}] &= \arg \max_{\mathbf{C}} \text{trace} \left\{ \text{Re} \left[\mathbf{Y}_t \mathbf{Y}_{t-1}^H \mathbf{X}_t^H \right. \right. \\ &\quad \left. \left. + \mathbf{Y}_{t-1} \mathbf{Y}_{t-2}^H \mathbf{X}_{t-1}^H + \mathbf{Y}_t \mathbf{Y}_{t-2}^H (\mathbf{X}_t \mathbf{X}_{t-1})^H \right] \right\}.\end{aligned}\quad (21)$$

4 THEORETICAL UPPER BOUND OF THE BEP

Suppose that the message \mathbf{X}_t is sent at each block. Since errors occur during transmission of the actual transmitted signal matrix \mathbf{S}_t due to channel fading and noise, after differential decoding, assume that the message \mathbf{E}_t in each block is detected. It follows that $\mathbf{E}_t \mathbf{E}_t^H = \mathbf{I}_{n_T}$, where \mathbf{I}_{n_T} is the $n_T \times n_T$ identity matrix. In order to measure the difference between \mathbf{X}_t and \mathbf{E}_t , we define $\mathbf{D}_t = \mathbf{E}_t \mathbf{X}_t^H$. Then, the matrix distance between \mathbf{X}_t and \mathbf{E}_t can be expressed as $\text{trace} \{ \text{Re} (\mathbf{I}_{n_T} - \mathbf{D}_t) \}$. When no error occurs, $\mathbf{D}_t = \mathbf{I}_{n_T}$, and thus $\text{trace} \{ \text{Re} (\mathbf{I}_{n_T} - \mathbf{D}_t) \} = 0$. Since $\mathbf{D}_t \mathbf{D}_t^H = \mathbf{I}_{n_T}$, matrix \mathbf{D}_t has the same orthogonal property as the message matrix \mathbf{X}_t and the actual transmitted signal matrix \mathbf{S}_t .

Recall that the DSM transmitter transmits the matrices $\mathbf{S}_t, \mathbf{S}_{t-1}, \dots, \mathbf{S}_{t-L+1}$ instead of directly transmitting the message matrices $\mathbf{X}_t, \mathbf{X}_{t-1}, \dots, \mathbf{X}_{t-L+2}$. Due to the influence of fading and noise, suppose that while $\mathbf{S}_t, \mathbf{S}_{t-1}, \dots, \mathbf{S}_{t-L+1}$ are transmitted, $\mathbf{Q}_t, \mathbf{Q}_{t-1}, \dots, \mathbf{Q}_{t-L+1}$ are actually received which causes that the differentially decoded message matrices $\mathbf{X}_t, \mathbf{X}_{t-1}, \dots, \mathbf{X}_{t-L+2}$ to become the error message matrices $\mathbf{E}_t, \mathbf{E}_{t-1}, \dots, \mathbf{E}_{t-L+2}$. Obviously, $\mathbf{Q}_t = \mathbf{E}_t \mathbf{Q}_{t-1} = \mathbf{E}_t, \mathbf{E}_{t-1}, \dots, \mathbf{E}_{t-L+2} \mathbf{S}_{t-L+1}$ and $\mathbf{Q}_t \mathbf{Q}_{t-1}^H = \mathbf{E}_t$.

Let η_c and η_e be the decision variables for transmission matrices \mathbf{X} and \mathbf{E} , respectively. Besides, let $\Pr(\mathbf{X} \rightarrow \mathbf{E} | \mathbf{X}, \mathbf{H})$ be the pairwise error probability of deciding \mathbf{E} when \mathbf{X} is transmitted for a given channel realization \mathbf{H} . Then, $\Pr(\mathbf{X} \rightarrow \mathbf{E} | \mathbf{X}, \mathbf{H})$ can be expressed as

$$\begin{aligned}\Pr(\mathbf{X} \rightarrow \mathbf{E} | \mathbf{X}, \mathbf{H}) &= P[(\xi_e - \xi_c) > 0 | \mathbf{X}, \mathbf{H}] \\ &= P \left(\left[\sum_{i=1}^{L-1} \sum_{j=0}^{i-1} \text{trace} \{ \text{Re} (\Phi_{t-i, t-j}) \} \right] > 0 | \mathbf{X}, \mathbf{H} \right),\end{aligned}\quad (22)$$

where

$$\eta_c = \text{trace} \left\{ \text{Re} \left[\sum_{i=1}^{L-1} \sum_{j=0}^{i-1} \mathbf{Y}_{t-j} \mathbf{Y}_{t-i}^H \mathbf{S}_{t-i} \mathbf{S}_{t-j}^H \right] \right\}, \quad (23)$$

$$\eta_e = \text{trace} \left\{ \text{Re} \left[\sum_{i=1}^{L-1} \sum_{j=0}^{i-1} \mathbf{Y}_{t-j} \mathbf{Y}_{t-i}^H \mathbf{Q}_{t-i} \mathbf{Q}_{t-j}^H \right] \right\}, \quad (24)$$

$$\Phi_{t-i, t-j} = \mathbf{Y}_{t-j} \mathbf{Y}_{t-i}^H \mathbf{Q}_{t-i} \mathbf{Q}_{t-j}^H - \mathbf{Y}_{t-j} \mathbf{Y}_{t-i}^H \mathbf{S}_{t-i} \mathbf{S}_{t-j}^H. \quad (25)$$

By using (23), (24) and (25), we have

$$\begin{aligned} & \text{trace} \{ \text{Re} (\Phi_{t-i,t-j}) \} \simeq \\ & \text{trace} \left\{ \text{Re} \left(\mathbf{H}_{t-i}^H \mathbf{H}_{t-j}^H (\mathbf{D}_{t-j} \mathbf{D}_{t-j-1} \dots \mathbf{D}_{t-i+1} - \mathbf{I}_{n_T}) \right) \right\} \\ & + \text{trace} \left\{ \text{Re} \left(\mathbf{S}_{t-j} \mathbf{H}_{t-j} \mathbf{N}_{t-i}^H \mathbf{Q}_{t-i} \mathbf{Q}_{t-j}^H \right. \right. \\ & \quad \left. \left. + \mathbf{N}_{t-j} \mathbf{H}_{t-i}^H \mathbf{S}_{t-i}^H \mathbf{Q}_{t-i} \mathbf{Q}_{t-j}^H \right. \right. \\ & \quad \left. \left. - \mathbf{S}_{t-j} \mathbf{H}_{t-j} \mathbf{N}_{t-i}^H \mathbf{S}_{t-i} \mathbf{S}_j^H \right. \right. \\ & \quad \left. \left. - \mathbf{N}_{t-j} \mathbf{H}_{t-i}^H \mathbf{S}_{t-i}^H \mathbf{S}_{t-i} \mathbf{S}_{t-j}^H \right) \right\}. \end{aligned} \quad (26)$$

Note that the second-order noise terms in (26) are ignored since they are quite small compared to other noise terms when SNR is large enough. Let

$$\begin{aligned} \Delta &= \sum_{i=1}^{L-1} \sum_{j=0}^{i-1} \text{trace} \{ \text{Re} (\Phi_{t-i,t-j}) \} \\ &= - \left(|h_1|^2 + |h_2|^2 + \dots + |h_{n_T}|^2 \right) \rho + \text{trace} \{ \text{Re} (\Theta) \}, \end{aligned} \quad (27)$$

where

$$\rho = \text{trace} \left\{ \text{Re} \left[\sum_{i=1}^{L-1} \sum_{j=0}^{i-1} (\mathbf{I}_{n_T} - \mathbf{D}_{t-j} \mathbf{D}_{t-j-1} \dots \mathbf{D}_{t-i+1}) \right] \right\}, \quad (28)$$

and

$$\begin{aligned} \Theta &= \sum_{i=1}^{L-1} \sum_{j=0}^{i-1} \left\{ \mathbf{S}_{t-j} \mathbf{H}_{t-j} \mathbf{N}_{t-i}^H \mathbf{Q}_{t-i} \mathbf{Q}_{t-j}^H \right. \\ & \quad \left. + \mathbf{N}_{t-j} \mathbf{H}_{t-i}^H \mathbf{C}_{t-i}^H \mathbf{Q}_{t-i} \mathbf{Q}_{t-j}^H \right. \\ & \quad \left. - \mathbf{S}_{t-j} \mathbf{H}_{t-j} \mathbf{N}_{t-i}^H \mathbf{S}_{t-i} \mathbf{S}_{t-j}^H \right. \\ & \quad \left. - \mathbf{N}_{t-j} \mathbf{H}_{t-i}^H \mathbf{S}_{t-i}^H \mathbf{S}_{t-i} \mathbf{S}_{t-j}^H \right\}. \end{aligned} \quad (29)$$

By substituting (26) and (27) into (22), we have

$$\begin{aligned} \Pr(\mathbf{X} \rightarrow \mathbf{E} | \mathbf{X}, \mathbf{H}) &= \Pr(\eta_e - \eta_c > 0 | \mathbf{X}, \mathbf{H}) \\ &= \Pr(\Delta > 0 | \mathbf{X}, \mathbf{H}). \end{aligned} \quad (30)$$

For given transmission matrices \mathbf{X} and \mathbf{E} , $\mathbf{Q}_t, \mathbf{Q}_{t-1}, \dots, \mathbf{Q}_{t-L+1}$, $\mathbf{S}_t, \mathbf{S}_{t-1}, \dots, \mathbf{S}_{t-L+1}$ and ρ can be considered as deterministic quantities. Therefore, we can easily show that $E[\text{trace} \{ \text{Re} (\Theta) \}] = 0$. Taking the expectation of both sides of (27) yields

$$\begin{aligned} E[\Delta] &= - \left(|h_1|^2 + |h_2|^2 + \dots + |h_{n_T}|^2 \right) E[\rho] \\ & \quad + E[\text{trace} \{ \text{Re} (\Theta) \}] \end{aligned} \quad (31)$$

$$= - \left(|h_1|^2 + |h_2|^2 + \dots + |h_{n_T}|^2 \right) \rho. \quad (32)$$

The computation of the variance of Δ is more complicated, because some terms in (29) are correlated although most of the terms are assumed to be mutually independent. Define

$$\begin{aligned} \Theta_1 &= \sum_{i=1}^{L-1} \sum_{j=0}^{i-1} \left\{ \mathbf{S}_{t-j} \mathbf{H}_{t-j} \mathbf{N}_{t-i}^H \mathbf{Q}_{t-i} \mathbf{Q}_{t-j}^H \right. \\ & \quad \left. + \mathbf{N}_{t-j} \mathbf{H}_{t-i}^H \mathbf{C}_{t-i}^H \mathbf{Q}_{t-i} \mathbf{Q}_{t-j}^H \right\}, \end{aligned} \quad (33)$$

$$\begin{aligned} \Theta_2 &= \sum_{i=1}^{L-1} \sum_{j=0}^{i-1} \left\{ \mathbf{S}_{t-j} \mathbf{H}_{t-j} \mathbf{N}_{t-i}^H \mathbf{S}_{t-i} \mathbf{S}_{t-j}^H \right. \\ & \quad \left. + \mathbf{N}_{t-j} \mathbf{H}_{t-i}^H \mathbf{C}_{t-i}^H \mathbf{S}_{t-i} \mathbf{S}_{t-j}^H \right\}. \end{aligned} \quad (34)$$

The variance of $\text{trace} \{ \text{Re} [\Theta] \}$ consists of three parts: the variance of $\text{trace} \{ \text{Re} [\Theta_1] \}$, the variance of $\text{trace} \{ \text{Re} [\Theta_2] \}$, and the cross-correlation between $\text{trace} \{ \text{Re} [\Theta_1] \}$ and $\text{trace} \{ \text{Re} [\Theta_2] \}$.

The variance of Δ can be obtained as

$$\begin{aligned} \text{Var}[\Delta] &= \text{Var}[\text{trace} \{ \text{Re} (\Theta) \}] \\ &= 4L(L-1) \left(|h_1|^2 + |h_2|^2 + \dots + |h_{n_T}|^2 \right) N_0 \\ & \quad + 2(L-2) N_0 \left(|h_1|^2 + |h_2|^2 + \dots + |h_{n_T}|^2 \right) \\ & \quad \text{trace} \left\{ \text{Re} \left[\sum_{i=0}^{L-1} \sum_{j=0}^{i-1} (\mathbf{D}_{t-j} \mathbf{D}_{t-j-1} \dots \mathbf{D}_{t-i+1} + \mathbf{I}_2) \right] \right\} \\ & \quad - 4(L-1) N_0 \left(|h_1|^2 + |h_2|^2 + \dots + |h_{n_T}|^2 \right) \\ & \quad \text{trace} \left\{ \text{Re} \left[\sum_{i=0}^{L-1} \sum_{j=0}^{i-1} (\mathbf{D}_{t-j} \mathbf{D}_{t-j-1} \dots \mathbf{D}_{t-i+1}) \right] \right\}, \end{aligned} \quad (35)$$

which can be simplified as

$$\text{Var}[\Delta] = 2L\rho \left(|h_1|^2 + |h_2|^2 + \dots + |h_{n_T}|^2 \right) N_0. \quad (36)$$

From (30), (31), and (36), we have

$$\begin{aligned} \Pr(\mathbf{X} \rightarrow \mathbf{E} | \mathbf{X}, \mathbf{H}) &= \Pr(\Delta > 0 | \mathbf{X}, \mathbf{H}) \\ &= Q \left(\sqrt{\gamma\rho \left(|h_1|^2 + |h_2|^2 + \dots + |h_{n_T}|^2 \right) / 2L} \right), \end{aligned} \quad (37)$$

where Q denotes the Gaussian tail function, and $\gamma = E_s/N_0$ is the SNR per symbol. By defining the instantaneous SNR as follows:

$$\gamma_b = \gamma \sum_{l=1}^{n_T} |h_l|^2, \quad (38)$$

and using the alternative form of the Gaussian Q-function [21], we can write

$$\begin{aligned} \Pr(\mathbf{X} \rightarrow \mathbf{E} | \mathbf{X}, \mathbf{H}) &= Q \left(\sqrt{(\rho/2L) \gamma_b} \right) \\ &= \frac{1}{\pi} \int_0^{\pi/2} \exp \left(-\frac{(\rho/2L) \gamma_b}{2 \sin^2 \Theta} \right) d\Theta. \end{aligned} \quad (39)$$

Averaging (39) over all realizations of the channel matrix \mathbf{H} gives us the PEP

$$\Pr(\mathbf{X} \rightarrow \mathbf{E} | \mathbf{X}) = \int_0^\infty Q \left(\sqrt{(\rho/2L) \gamma_b} \right) p(\gamma_b) d\gamma_b, \quad (40)$$

where $p(\gamma_b) = \frac{1}{(D-1)! \gamma^D} \gamma_b^{D-1} e^{-\frac{\gamma_b}{\gamma}}$ is the pdf of γ_b and D is the number of diversity channels [22].

Let \mathbf{u} represent a sequence with q information bits and $\hat{\mathbf{u}}$ denotes an error sequence with the same number of information bits. The probability of error P_b of the

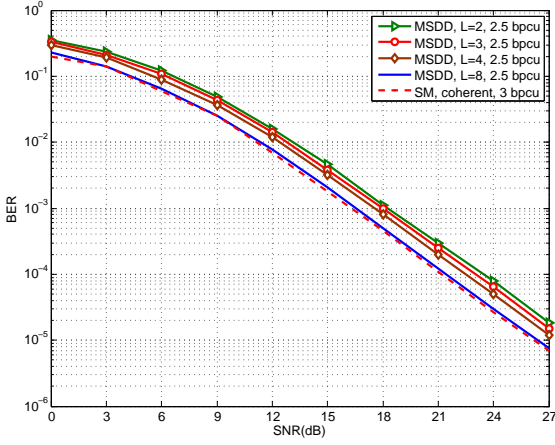


Figure 3. BER versus SNR for different lengths of observation interval for DSM system at 2.5 bpcu, $(n_T, n_R) = (2, 2)$.

proposed detection method for DSM system is union-bounded by [23, 24]:

$$P_b \leq \frac{1}{2q} \sum_{\mathbf{x} \neq \mathbf{E}} \Pr(\mathbf{X} \rightarrow \mathbf{E} | \mathbf{X}) \cdot w(\mathbf{u}, \hat{\mathbf{u}}), \quad (41)$$

where $w(\mathbf{u}, \hat{\mathbf{u}})$ is the Hamming distance between sequences \mathbf{u} and $\hat{\mathbf{u}}$. The PEP $\Pr(\mathbf{X} \rightarrow \mathbf{E} | \mathbf{X})$ is given by (40).

5 PERFORMANCE EVALUATIONS

In this section, Monte Carlo simulations and the theoretical upper bound are used to study the BER performance of the proposed detection method for DSM system. Simulations are carried out over the slow, flat Rayleigh fading channel. The channel is assumed to be constant during the observation interval, but changes independently from interval to interval. The channel state information is assumed unavailable at the receiver, and ML sequence detection is applied to all systems under consideration.

Using the decision statistic in (19), the DSM receiver can detect blocks of differentially encoded signals by observing over intervals of different lengths. Figure 3 shows the BER performance curves of DSM at spectral efficiency of 2.5 bpcu using QPSK signal constellation and various observation intervals. Figure 3 also compares the BER performance of the DSM scheme and the coherent SM scheme under the same modulation order. The simulations are performed with $n_T = 2$, $n_R = 2$. The curve for $L = 2$ corresponds to the DSM scheme suggested in [14] and [25]. Indeed, this curve of DSM matches well with the results in the reference and the performance degradation of DSM over SM is closer to 3 dB. Notice from the figure that when the observation interval is increased, the BER performance of MSDD approaches closer to that of the coherent detection. Specifically, when $L = 8$ the proposed MSDD achieves the same BER performance as that of the coherent detection. However, it is worth noting that for the same signal constellation, the spectral efficiencies of SM and DSM are not the same. For the considered

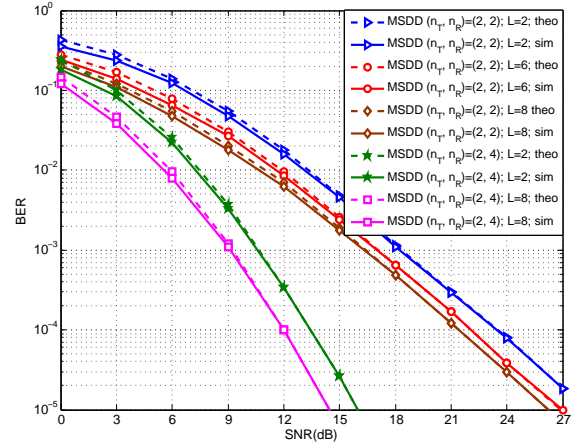


Figure 4. Analysis and simulation results for MSDD of DSM at 2.5 bpcu.

case of QPSK the spectral efficiency of DSM is 2.5 bpcu while that of SM is 3 bpcu. It is noted again from (19) that the computational complexity of MSDD increases exponentially with L . Therefore, a careful selection of the observation interval L will help to balance the performance improvement with additional complexity.

In Figure 4, the derived theoretical upper bounds for the DSM BEP is validated by simulation results for two arbitrary cases of $L = 2$ and $L = 6$. It can be seen that the upper bounds fit well with the simulation curves, particularly at high SNR. Therefore, this tight upper bound can be used for evaluating the performance of DSM in the high SNR region.

6 CONCLUSION

A multiple-symbol differential detector is proposed for differential spatial modulation, where neither the transmitters nor the receivers know the channel state information. A generalized decision metric for an observation interval of L blocks is derived. In addition, a theoretical upper bound for the bit error probability for the DSM system that utilizes the proposed detector is provided. It is shown that the upper bounds fit well with the simulation results, particularly in the high SNR region. It is also demonstrated by computer simulation that the MSDD allows the DSM system to improve its BER performance. The longer observation interval is used, the larger performance improvement is achieved. The penalty is, however, that detection complexity grows exponentially with the observation interval.

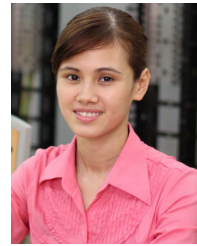
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